Syllabus for comprehensive examination for specialization in Design

Mechanics

1. *Vector Mechanics for Engineers – Statics by Ferdinand P. Beer and E. Russell Johnston Jr.*
2. *Vector Mechanics for Engineers-Dynamics by Ferdinand P. Beer and E. Russell Johnston Jr.*
3. *Engineering Mechanics of Solids by Egor P. Popov*
4. *Mechanics of Materials by Ferdinand P. Beer, E. Russell Johnston, John T. Dewolf and David F. Mazurek*

STATICS

1. Equilibrium of rigid bodies [**Ref. 1, Chapter 4**]
    a. Free-body diagrams
    b. Equilibrium of rigid bodies in two and three dimensions

2. Centroids, Centers of gravity and moment of inertia
    a. Centroids and centers of gravity and their determination [**Ref. 1, Chapter 5**]
    b. Moment of inertia of areas and masses and their determination [**Ref. 1, Chapter 9**]

3. Forces in beams [**Ref. 1, Chapter 7**]
    a. Shear force and bending moment diagrams
    b. Relations among load, shear force and bending moment

4. Friction [**Ref. 1, Chapter 8**]
    a. Laws of dry friction
    b. Angle of friction
    c. Problems involving friction such as blocks on inclined planes
    d. Wheel friction and rolling resistance

DYNAMICS

1. Kinematics of Rigid Bodies [**Ref. 2, Chapter 15**]
    a. Translation, rotation and general plane motion
    b. Instantaneous center of rotation in plane motion
    c. Absolute and relative acceleration in general plane motion
    d. Motion relative to rotating reference frames

2. Kinetics of general plane motion [**Ref. 2, Chapter 16**]
    a. Equations of motion
    b. Angular momentum
    c. D'Alembert's Principle
    d. Motion constrained to a plane

3. Energy and momentum conservation for plane motion [**Ref. 2, Chapter 17**]
    a. Kinetic energy of rigid body in plane motion
    b. Conservation of energy
    c. Principles of Impulse and momentum
    d. Conservation of angular momentum

Strength of materials

1. Stress and deformation analysis of axially loaded bars [**Ref. 3, Chapter 2**]
    a. Analysis of normal and shear stresses
    b. Thermal strains
    c. Solution of statically indeterminate problems
    d. Calculations of strain energy

2. Equilibrium equations and generalized Hooke's law (3D) [**Ref. 3, Chapter 3**]
    a. Static equilibrium equations
    b. Stress-Strain relations for isotropic materials
    c. Relations between E, ν, G and K for isotropic materials
    d. Stress analysis of pressure vessels with thin walls

3. Torsion of rods [**Ref. 3, Chapter 4 (PART A only)**]
    a. Torsion formula and its underlying assumptions
    b. Determination of stresses, strains and twist of statically determinate and indeterminate members
    c. Calculations of strain energy

4. Bending of beams (Pure Bending) [**Ref. 3, Chapter 6**]
    a. Basic assumptions
    b. Elastic flexure formula and its applications
    c. Calculations of strain energy

5. Transformation of stress and strain [**Ref. 3, Chapter 8**]
    a. Transformation of stress and strains for two dimensional problems
    b. Mohr's circle for stress and strains for two dimensional problems
    c. Concepts of Principal stresses

6. Deflection of beams [**Ref. 3, Chapter 10**]
    a. Moment-Curvature relations
    b. Governing differential equation, appropriate boundary conditions and their solution
    c. Stress, strain and deflection analysis of statically indeterminate beams

Syllabus for Comprehensive examination for specialization in Design

Mechanics and Materials

1. *Vector Mechanics for Engineers – Statics by Ferdinand P. Beer and E. Russell Johnston Jr.*
2. *Engineering Mechanics of Solids by Egor P. Popov*
3. *Mechanics of Materials by Ferdinand P. Beer, E. Russell Johnston, John T. Dewolf and David F. Mazurek*
4. *Materials Science and Engineering an Introduction, W. D. Callister, 8th Edition*
5. *Machine Design: An integrated approach, Robert L. Norton, 3rd edition*

STATICS

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    a. Centroids and centers of gravity and their determination [**Ref. 1, Chapter 5**]
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4. Friction [**Ref. 1, Chapter 8**]
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    c. Problems involving friction such as blocks on inclined planes
    d. Wheel friction and rolling resistance

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Materials science

1. Stress-strain curve for mild steel and mechanisms of deformation [**Ref. 4, 6.1-6.9**]
    a. Elastic deformation
    b. Strain hardening
    c. Relations between true stress-strain and engineering stress-strain
    d. Atomistic mechanisms for upper and lower yield points, necking and fracture

2. Iron Iron-carbide equilibrium diagram [**Ref. 4, 9.18-9.20, 10.5 – 10.6**]
    a. Various phases, temperatures and reactions
    b. Concept of tie-line to determine compositions and phase amounts
    c. Microstructures of hypo, hyper and eutectoid steels
    d. Isothermal transformation diagrams
    e. Time Temperature Transformation diagrams
    f. Effect of alloying elements on mechanical properties of steel

3. Failure theories [**Ref. 5, 5.1-5.2**]
    a. Fundamentals and applications of the following failure theories applied to cases with general states of stresses
        i. Distortion energy theory
        ii. Maximum shear stress theory

b. Fatigue failure [**Ref.5, 6.1-6.7**]
    i. S-N diagram
    ii. Endurance limit
    iii. Effects of mean stress on fatigue life
        1. Goodman, Soderberg lines and Gerber parabola

Vibration: Comprehensive Written Exam Syllabus

1. **Single degree-of-freedom model**

   (a) Free undamped and damped vibration: Solving second order differential equations for combinations of displacement and velocity initial conditions ([1], Ch 2, Sec. 2.1-2.3)

   (b) Viscous damping: Logarithmic decrement, Structural damping and Coulomb friction ([1], Ch 2, Sec. 2.4 and 2.5, Ch 3 Sec. 3.9)

   (c) Equivalent Viscous damping ([1], Ch 3, Sec. 3.7-3.10)

   (d) Forced vibration: Steady-state response to sinusoidal excitation; dynamic magnification factor amplitude and phase as a function of excitation frequency ([1], Ch 3, Sec. 3.1); base excitation or rotating unbalance excitation; displacement response amplitude and frequency variation with excitation frequency ([1], Ch 3, Sec. 3.3, 3.4 and 3.6); vibration isolation- generation of force transmissibility relations and design criteria for isolation ([1], Ch 3, Sec. 3.2); periodic excitation - use of superposition principle to obtain the response to multiple harmonic inputs ([2], Ch 3 Sec 3.9)

   (e) Transient excitation: Concept of impulse response and convolution integral ([1], Ch 4, Sec. 4.1, 4.2); Use of Duhamel's integral to obtain response to step and pulse type excitations ([1], Ch 4 Sec 4.4); concept of shock response spectrum generation and shock isoation ([1], Ch 4, Sec 4.5 and 4.6)

2. **Multi degree-of-freedom models**

   (a) Deriving equations of motion for complex models

   (b) Concept of mode shapes and associated mathematical properties- Generalized eigenvalue problem solution for natural frequencies and mode shapes, orthogonality property of mode shapes with respect to mass and stiffness matrices([2] Ch 7 Sec. 7.6, 7.7)

   (c) Proportional or Rayleigh damping - Generating damping matrices from stiffness and mass matrices; diagonalizing damping matrices; generating the coefficients for Rayleigh damping model ([2] Ch 7 Sec. 7.15)

   (d) Use of modal superposition to obtain free and forced vibration response ([2] Ch 7, Sec. 7.9, 7.10 and 7.15)

   (e) Vibration absorber application ([1] Ch 5 Sec. 5.13 and 5.14)

3. **Continuous system vibration models**

   (a) Derivation of equations of motion for transverse vibration of strings - Natural frequencies and mode shapes for different boundary conditions; use of mode shapes to find free vibration response ([2] Ch 8 Sec. 8.1, 8.4, 8.5, 8.9 and 8.10)

   (b) Derivation of equations of motion for beam bending vibrations- Obtain natural frequency and mode shape expressions for different boundary conditions; free and forced vibration of beams using modal superposition ([2] Ch 8 Sec. 8.3, 8.4, 8.5, 8.9 and 8.10)

Books

1. W. T. Thomson, M. D. Dahleh and C. Padmanabhan, 2008, *Theory of Vibration with Applications*, 5th Edition, Pearson Education India: New Delhi.

2. L. Meirovitch, 2001, *Fundamentals of Vibrations*, McGraw-Hill International Edition: Singapore.

Elasticity

Module 1:

Compute eigenvalues, eigenvectors. Define an orthogonal transformation. Compute the directional derivative at a point. Compute the gradient, divergence and the curl of a vector and a second order tensor. Express tensorial quantities in indicial notation and perform operations using them. State and apply the Gauss divergence theorem. **Chapter 1 in [1]**

Module 2:

Calculate the deformation gradient. Decompose it into a rotation and a symmetric tensor. Explain the physical meaning of determinant of F. Define Green-St Venant and Almansi Hamel strain (strain measures for finite deformation). Provide a physical interpretation for them. Relate the small strain to the finite deformation strains. Calculate Cauchy stresses and find the traction on any plane through a point given the stress at that point (Mohr Circle). State and justify the need for compatibility relations. Show that they are necessary and sufficient to obtain unique displacement fields. State and derive the balance laws (mass, linear momentum, angular momentum and energy). **Chapters 2,3,4,5 in [1]**

Module 3:

Obtain the constitutive relation for an isotropic linearized elastic solids. Perform the reduction from 81 constants to 21 constants in the most general case and further reduce to nine, five or two constants by using appropriate symmetry restrictions. Obtain bounds on elastic constants. Formulate boundary value problems. Distinguish between plane strain and plane stress. Apply Airy stress function (rectangular and cylindrical coordinates) to solve plane problems. Solve a few boundary value problems: cylindrical pressure vessel, torsion of prismatic cylinders, and circular hole in an infinite plate. **Chapters 4, 5, 7, 8 and 9 in [2]**

References

1) Schaum's Outline of Continuum Mechanics (Schaum's Outline Series), George Mase, 1st Edition, 2005, Tata McGraw-Hill, Delhi.
2) Elasticity – Theory, Applications and Numerics, Martin H. Sadd, 2nd Edition, 2012, Elsevier India, New Delhi.

Theory of Mechanisms – Comprehensive Exam Syllabus

Review of kinematics fundamentals – degrees of freedom, mobility, classification of joints and links, Grashof's criterion, kinematic diagrams of real-life mechanisms, inversions of a mechanism. Number synthesis. Kinematic equivalence of mechanisms ***[Ghosh and Mallik– Chap. 1].***

Graphical synthesis of mechanisms for motion, function and path generation, quick-return motion, driver dyads ***[Norton–Chap. 3]***.

Review of complex numbers and its application to mechanism design and analysis. Analytical synthesis of mechanisms: precision points using Chebyshev spacing, Dyad or standard form for 3- and 4-point synthesis, Burmester theory, M- and K-curves of Burmester points ***[Erdman and Sandor, Vol. II, Sec. 2.2, 2.12-2.19, 2.21-2.23, 3.1-3.5]***.

Kinematic synthesis of other planar mechanisms: coupler curves and cognates ***[Hartenberg and Denavit, Sec 6-1, 6-3, 6-4***].

Graphical and analytical methods for *k*inematic analysis of planar mechanisms, including kinematically complex mechanisms ***[Norton, Chaps. 6 and 7, Ghosh and Mallik, Sec. 2.3-2.8]***.

Force analysis of mechanisms – Newton-Euler method, Mechanical advantage ***[Norton Sec. 11.1–11.9].***

Introduction to the analysis and synthesis of spatial mechanisms: kinematic diagram, mobility, transformation of points and vectors between reference frames ***[Ghosh and Mallik, Sec. 5.1,5.2].***

Books

[1] Kinematics and Dynamics of Machinery, Robert L. Norton

[2] Advanced Mechanism Design, Vol. II, Erdman and Sandor

[3] Kinematic Synthesis of Linkages, Hartenberg and Denavit (freely available online)

[4] Theory of Mechanisms and Machines, Ghosh and Mallik

Finite Element Method

Module 1:
Mathematical models in one dimension, application to discrete systems, conceptualization, boundary value problems, basis functions, boundary conditions (Dirichlet, Neumann and Robin boundary conditions), FEM in one dimensions, polynomial space, Treatment of boundary conditions, Post-processing.

Module 2:
Strong and weak forms for scalar and vector problems, equivalence between strong and weak forms, two point boundary value problems with generalized boundary conditions, Finite element spaces, Lagrangian and Hermite interpolants, Approximation of Trial solutions, Gauß quadrature, Isoparametric elements, Mapping in two and three dimensions, Jacobian, Linear elements (triangular/quadrilateral in two dimensions & tetrahedral/hexahedral in three dimensions), serendipity elements, convergence, boundary force matrix.

Module 3:
Application to problems in engineering: plane elasticity, three dimensional elasticity, transient heat transfer problems, potential flow, coupled problems.

Suggested References

1. J Fish and T Belytschko, *A first course on finite elements*, Wiley, 2007.
2. B Szabó and I Babuška, *Introduction to finite element analysis*, Wiley, 2011.
3. Thomas JR Hughes, *The Finite Element Method: Linear static and dynamic finite element analysis*, Prentice Hall, New Jersey, 1987.

Syllabus for Fracture Mechanics and Failure Analysis

Need for fracture based design; design methodologies – safe-life, fail-safe, damage tolerance design; Stress Concentration Factors; Stress Intensity Factors; Griffith's energy balance criteria; Dugdale's correction to Griffith's criteria; Compliance-Energy Release correlation.

Stress Field ahead of crack tip – Review of theory of elasticity, Airy's stress Function, Westergaard's solution for crack tip stress, displacement; Modes of fracture; Relationship between Energy Release Rate (G), Resistance to cracking (R) and Stress Intensity Factor (K). Plane stress, plane strain conditions at crack tip and its relation to crack driving force.

Crack tip plastic zone, Implications of crack tip plasticity, Stable crack extension, R-Curve Concept, Elastic-plastic fracture, J-Integral concept. Crack arrest methodology. Dynamic fracture. Fracture testing methodologies. Application of fracture mechanics to Fatigue crack growth. Crack closure mechanisms.

Reference:

[1] Lecture Notes on Fracture Mechanics, NAL – Dr. K N Raju, Vol 1, 1979 (also available as STC course lecture by T S Ramamurthi, IISc, Bangalore).

[2] Fracture Mechanics: Fundamentals and Application – T. L. Anderson, CRC Press, 3/e, 2004. Chapters 2 and 3

[3] ASTM Annual book of standards 3.01 for experimental methods of fracture parameter determination ASTM E-399, E-647, E-1820.

[4] K. Ramesh, NPTEL lecture notes on Fracture Mechanics.

<u>Failure Analysis</u>

Examples of case studies in failure analysis (Ch 1), examination and reporting procedures (Ch 5), brittle and ductile fractures (Ch 6), creep failure (Ch 9), fatigue failure (Ch 10), environmental effects (Ch 12), basics of methods of flaw detection (visual, dye penetrant, magnetic particle testing, eddy current testing, ultrasonic testing, radiographic testing, acoustic emission testing)(Ch 13).

Reference:

[1] Metal Failures: Mechanisms, Analysis and Prevention by Arthur J McEvily, Wiley-Interscience Publication, 2002.

Comprehensive examination syllabus

Applied Mathematics

Machine Design Section
Department of Mechanical Engineering
IIT Madras

1 Linear Algebra [1, 2]

- **Solution of linear algebraic equations of the form** $[A]_{m\times n}\{x\}_{n\times 1} = \{b\}_{m\times 1}$
  - Identifying $\{x\}$ and $\{b\}$ as elements of a vectors space $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively.
  - Definition of a vector space
  - Examples of vector spaces such as $\mathbb{R}^n$, $\mathbb{C}^n$, $\mathbb{R}^{m\times n}$, Polynomials of degree n, Continuous functions, Periodic functions with period T, etc.
  - Linear independence / dependence of vectors in a vector space
  - Basis of a vector space, Complex exponentials e^{int} as basis for 2π periodic functions
  - Linear transformation between vector spaces, Change of basis as a linear transformation
  - Matrix representation of linear transformation
  - Identifying $Ax = b$ as a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.
  - Column space and row space of a matrix
  - Subspace of a vector space
  - Range and Null space for a linear transformation, determination of the same using elementary row operations.
  - Finding solution of $Ax = b$ using elementary row operations
  - Rank-nullity theorem
  - Existence and uniqueness of solutions for $[A]_{m\times n}\{x\}_{n\times 1} = \{b\}_{m\times 1}$.
  - Inter-relationship of existence / uniqueness of solutions of homogeneous and non-homogeneous system of equations.
- **Eigenvalue Problem**
  - Definition of eigen value problem
  - Non-uniqueness of eigen vectors, normalization conditions for eigen vectors
  - Relationship between the homogeneous solution and eigen value problem, Significance of condition number in numerical calculations
  - Invariants of a Linear transformation / matrix on change of basis
  - Diagonalization of matrices and its significance in solving system of equations

- Diagonalization as a change of basis to the eigen basis
- Conditions for diagonalizability of matrix

- **Orthogonality & Symmetric Matrices**
  - Inner product as a generalization of perependicularity conditions.
  - Inner product of two vectors, 2-Norm of a vector as inner product of the vector with itself
  - Orthogonal basis for a vector space and its significance, Gram-Schmidt ortho-normalization
  - Fourier series as an orthogonal basis for the vector space of periodic functions
  - Orthogonal subspaces, Direct sum decomposition of subspaces
  - Given $[A]_{m\times n}$ show Null Space $(A) \oplus \text{Range}(A^T) = \mathbb{R}^n$ and Null Space $(A^T) \oplus \text{Range}(A) = \mathbb{R}^m$
  - Finding the minimum norm solution in case of non-unique solution for $[A]_{m\times n}\{x\}_{n\times 1} = \{b\}_{m\times 1}$
  - Finding the least square solution in case of non-existing solution for $[A]_{m\times n}\{x\}_{n\times 1} = \{b\}_{m\times 1}$
  - Existence of real eigen values for symmetric matrices
  - Orthogonality of eigen vectors for symmetric matrices
  - Diagonalizability of symmetric matrices

2 Differential Equations

- Special Methods for ODEs [3]
  - Separable first order ODEs
  - Exact Differential Equations and Integrating factors
  - Second order constant coefficient ODE, Phase Portrait

- General methods for constant cofficient linear ODEs [3, 1, 4]
  - Higher order Linear constant coefficient ODEs
  - Homogeneous and Non-homogeneous equation, General ODE solution as a superposition of homogeneous and Nonhomogenous solution
  - Reduction of homogeneous constant coefficient linear ODEs (or arbitrary order) to state-space form $\{\dot{x}\} = [A]\{x\}$
  - Solving the above system for diagonalizable/ non-diagonalizable A, reduction to exponential functions
  - Laplace and Fourier transform methods in solving general and particular solution for ODEs

- Partial Differential Equation
  - Writing the following equations in cartesian coordinates (no derivation or solving) (1) 1D transient heat conduction (2) 2D steady state heat conduction (3) 2D transient heat conduction (4) 1D wave equation (5) 2D wave equation

- Linear PDEs and superposition principle, Initial and Boundary value problems
- Solution using separation of variable for 1D wave equation for different initial and boundary conditions, Use of Fourier series
- D'Alembert's Solution of 1D wave equation
- Solution for 1D transient heat equation using Separation of variables and Fourier Transform methods
- Distinction between solution of wave equation and heat equation

References

1. G. Strang, Linear Algebra and its Applications, Thomson/ Brooks Cole. Also the related ocw videos.
2. R. V. Rao, Linear Algebra and its Applications, NPTEL video lectures
3. E. Kreyszig, Advanced Engineering Mathematics, John Wiley.
4. G. Strang, Introduction to Applied Mathematics, Wellesley Cambridge Press. Also the related ocw videos.